

Derivatives

Chain Rule

1. Find the derivative of $f(x) = \cos^3\left(\frac{1}{3x+1}\right)$.
2. Find the derivative of $f(x) = x^5 \cos(3x)$.
3. Find the derivative of $\sqrt{\sin(2x)}$.
4. Find the derivative of $\sin(\sqrt{x})$.
5. Find the derivative of $\cot(3x^2)$.

Inverse Functions

6. Find the derivative of $\operatorname{arccot}(x)$.
7. Find the derivative of $\arcsin(x)$.
8. Let $f(x) = xe^x$ and let g be the inverse function of f . Given that $f(1) = e$, find $g'(e)$.
9. Let $f(x) = x^5 + 4x^3$ and let g be the inverse function of f . Given that $f(1) = 5$, find $g'(5)$.
10. Let $f(x) = x^5 + x + 1$ and let g be the inverse function of f . Find the derivative of g at $(3, 1)$.

Implicit Differentiation

11. Find the derivative of the function $y^2(6 - x) = x^3$ at $(3, 3)$.
12. Find $\frac{dy}{dx}$ given that $\frac{1}{y} + \frac{1}{x^2} = 1$.
13. Let $y^2 = x^2(x - 1)$. At what points is $\frac{dy}{dx}$ not defined?
14. Find $\frac{dy}{dx}$ if $\ln(xy) = e^y$.
15. Find when the curve $x^4 = 2x^2 - y^2$ has a horizontal derivative.

L'Hopital's Rule

16. Find $\lim_{x \rightarrow \infty} \sqrt{2x+1} - \sqrt{x+1}$.
17. Find $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2-1}}$.
18. Find $\lim_{x \rightarrow \infty} x^{1/x}$.
19. Find $\lim_{x \rightarrow 0^+} x^x$.
20. Find $\lim_{x \rightarrow 0} \frac{\cos(x) - 1 + \sin(x)^2/2}{x^4}$.

Application

Optimization

21. Find the area of the smallest triangle formed by the x axis, y axis, and a line that goes through the point $(1, 1)$.
22. Find the largest rectangle that can be inscribed into a semicircle of radius 2 so that one side of the rectangle is part of the diameter of the semicircle.
23. Find the point on the curve $y = 1 - \sqrt{x}$ closest to $(1, 1)$.
24. A rectangle is inscribed under the curve $\sin x$ for $0 \leq x \leq \pi$. This rectangle has two vertices on the curve and one side on the x axis. What is the maximum possible area of such a rectangle.
25. What is the point on $y = e^x$ closest to $(1, 0)$?

Related Rates

26. A ball of light is bobbing up and down and whose position is given at a time t by $4 + \sin(2t)$. A man who is $2m$ tall is standing $10m$ away. How fast is the length of his shadow changing when $t = 0$?
27. A conical cup that is $6cm$ wide at the top and $5cm$ tall is filled with water is punctured at the bottom and water is coming out at a rate of $10^{-6}\pi m^3/s$. Initially, the cup is filled. How fast is the height of the water changing when the height is $3cm$?
28. Sand is being dumped in a conical pile whose radius and height always remain the same. If the sand is being dumped in at a rate of $2\pi m^3/hr$, how fast is the height of the sand changing when the pile is $5cm$ tall?

29. A kite is flying at a current altitude of $100m$. The kite slowly flies further and further away as the string length increases at a rate of $2cm/s$. Assuming the altitude does not change, how fast horizontally is the kite moving when the angle the string forms with the ground is $\pi/4$?
30. A ladder $13m$ tall is lying against a wall. The bottom of the ladder is pulled out at a rate of $10cm/s$. How fast is the area of the triangle formed by the ladder, wall, and floor changing when the bottom of the ladder is $5m$ away from the wall?

Taylor Series

31. Use the third order approximation to find $\sin(0.5)$.
32. Approximate $\sqrt{99}$ using a quadratic regression.
33. Use the second order approximation to find $\ln 1.01$.
34. Use the second order approximation to $\sqrt[3]{8.1}$.
35. Use the quintic order approximation to find e .

Newton's Method

36. Use Newton's method once to approximate $\sqrt[3]{8.1}$.
37. Approximate $\sqrt{99}$ using Newton's method once.
38. Find the critical points of $g(x) = \sin(x) - x^2$
39. Find the unique solution to $(\pi - 2x) \cos(x) = 2 \sin(x)$ on the interval $[0, 1]$ using Newton's method with an initial guess of $x = \frac{\pi}{4}$.
40. Find when $\cos(x) = x$ using Newton's method and an initial guess of $x = \frac{\pi}{6}$.

Functions

Domain/Range

41. Find the domain of $y = \sqrt{9 - (2x + 3)^2}$.
42. Find the domain of $y = \frac{1}{\sqrt{3-x}}$.
43. Find the domain and range of $2 - \arccos(3x + 2)$.
44. Find the domain of $\frac{\ln(x+3)}{\sqrt{2-x}}$.
45. Find the domain of $\sqrt{\frac{3-x}{1-x}}$.

Inverse Functions

46. Find the inverse of $f(x) = \frac{-2}{x} - 1$.
47. Find the inverse of $\frac{4+\sqrt{3x}}{5}$.
48. Find the inverse to x^2 on $(-\infty, 0]$.
49. Find the inverse to e^{2x+3} .
50. Find the inverse to $-\sqrt{\ln x}$.

Graphing

51. Sketch the graph of $f(x) = \frac{x}{x^2 + 1}$.
52. Sketch the graph of $f(x) = x + \frac{1}{x-1}$.
53. Sketch the graph of $f(x) = 3 - 15x - 6x^2 + x^3$.
54. Sketch the graph of $f(x) = \frac{x-1}{x+1}$.
55. Sketch the graph of $f(x) = e^x + 2e^{-x}$.